

Notes for

B.Sc. Part-II

Paper - 3rd

- By

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### Dispersion in metals.

We know that conductivity  $\sigma$ , dielectric constant  $\epsilon$  and permeability  $\mu$  are not true constants of a medium, but these are depend on the frequency  $\omega$  of the field.

We also know that the response of a dielectric, medium + an external electromagnetic field is largely determined by the behaviour of electrons that are bound to the atomic nuclei by quasi elastic forces in a conducting medium, like metals, unlike in a dielectric not all the electrons are bound to the atoms. Some move between the molecules and are said to be free electrons. These are distinguished from the other electrons which are bound to the atoms, just as in a dielectric.

In the absence of an external electromagnetic field, the free electrons move in a random manner and hence they do not give rise to a net current flow. When an external field is applied the free electrons acquire an additional velocity and their motion becomes more orderly, even though occasionally they collide with the atoms. This orderly motion of the electrons give rise to the induced current flow.

The damping force is proportional and opposite in direction to the velocity of a model electron that represents the average behaviour of the whole set of electrons. Therefore the equation of motion of this model electron in an electric field  $\vec{E}$  is given by -

$$m \frac{d^2\vec{r}}{dt^2} + m\beta \frac{d\vec{r}}{dt} = e\vec{E} \quad \rightarrow \textcircled{1} \rightarrow$$

(P.T.O)

→ (from Page-1)

→ where  $m$  is the mass,  $e$  the charge of the electron and  $\beta$  the damping constant referred to unit mass and  $\vec{E}$  is a microscopic electric field which is believed to represent more closely the field that acts on a free electron in a conductor.

In order to understand the meaning of the damping constant  $\beta$  in equation ①, let us consider the case where no electric field is present i.e.  $\vec{E} = 0$ .

∴ Equation ① reduces to -

$$\frac{d^2\vec{r}}{dt^2} + \beta \frac{d\vec{r}}{dt} = 0 \quad \rightarrow ②$$

The solution of this equation ② is given by -

$$\vec{r} = \vec{r}_0 - \frac{1}{\beta} \vec{v}_0 e^{-\beta t} \quad \therefore \frac{d\vec{r}}{dt} = \vec{v} = \vec{v}_0 e^{-\beta t} \quad \rightarrow ③$$

where  $\vec{v}_0$  is the initial velocity and  $\vec{v}$ , the velocity of the electron at the instant  $t$ . In this case we see that the model electron starting with the velocity  $\vec{v}_0$  is slowed down in an exponential way with  $\beta$  as decay constant. The time  ~~$T$~~   $T = \frac{1}{\beta}$  is called the decay time or the relaxation time. It is of the order of  $10^{-14}$  sec.

Now let us assume that the electric field is time harmonic and is given by -  $\vec{E} = \vec{E}_0 e^{i\omega t} \quad \rightarrow ④$

thus the solution of equation ④ is the sum of two terms, one representing the decaying motion and the other representing a periodic motion.

$$\vec{r} = \frac{e}{m(\omega^2 + i\beta\omega)} \vec{E} \quad \rightarrow ⑤$$

This Periodic gives rise to a current in the medium. If there are  $N$  free electrons per unit volume then conductivity  $\sigma$  is given by  $\sigma = \frac{Ne^2}{m(\beta - i\beta)}$   $\rightarrow ⑥$ .

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