

Notes for

B.Sc. Part-II

- By Dr. Bharat Singh

Dept. of Physics

Paper-B ref. : Double refraction in Crystals

Let us consider a plane incident from vacuum on a surface Σ of an isotropic medium. This gives rise to a transmitted and reflected field. It holds an argument as in the case of an isotropic medium to find the direction of propagation of the disturbance in the crystal.

Let \vec{s} be the unit wave normal of the incident wave and \vec{s}' that of the transmitted wave. In general two waves are transmitted so that there are two possible values of \vec{s}' . The field vectors of the incident wave and of transmitted wave are functions of $\left\{ t - \frac{\vec{r} \cdot \vec{s}}{c} \right\}$ and $\left\{ t - \frac{\vec{r} \cdot \vec{s}'}{v'} \right\}$ respectively.

The continuity of the field across the boundary demands that for any point \vec{r} on the plane and for all time t .

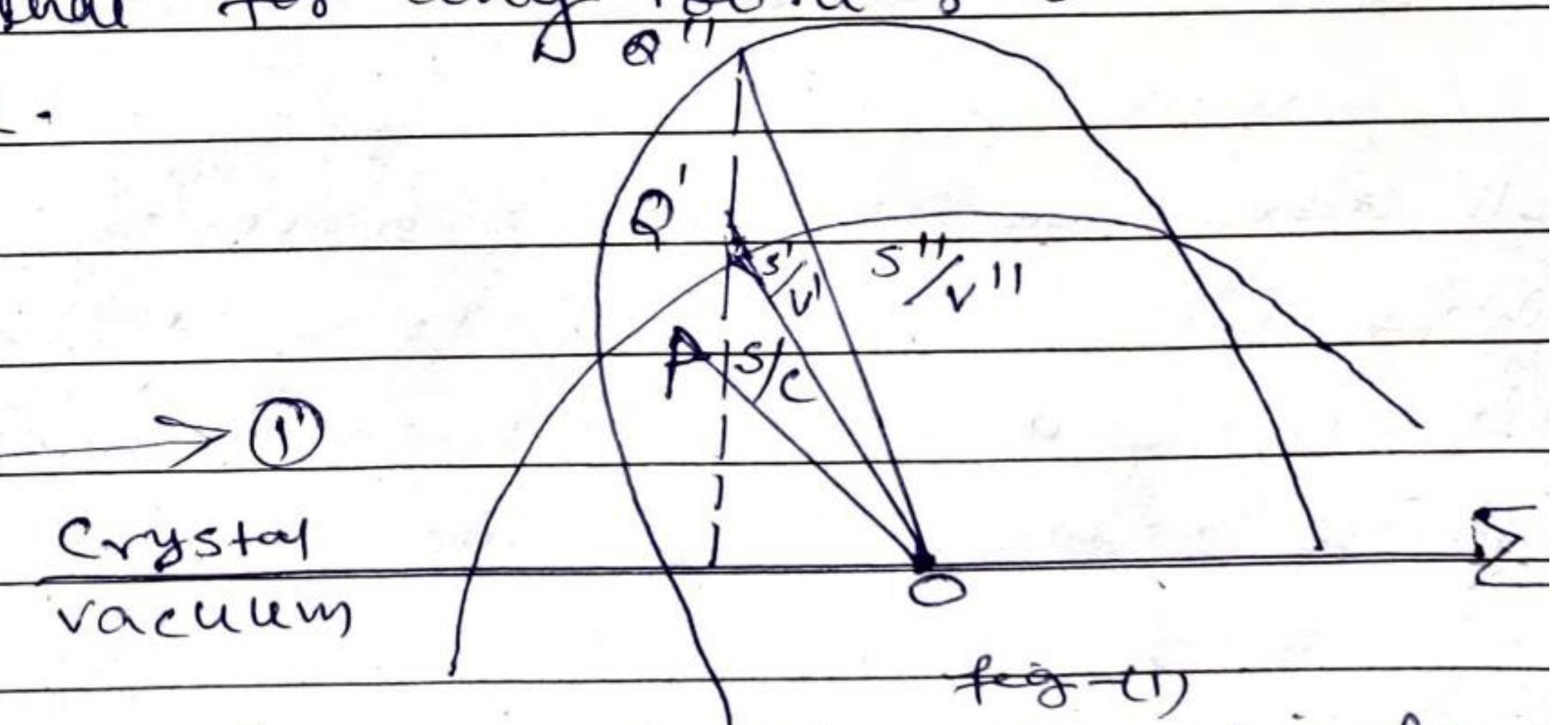
$$t - \frac{\vec{r} \cdot \vec{s}}{c} = t - \frac{\vec{r} \cdot \vec{s}'}{v'}$$

$$\text{or, } \vec{r} \left\{ \frac{\vec{s}'}{v'} - \frac{\vec{s}}{c} \right\} = 0 \quad \rightarrow (1)$$

Hence the factor

$$\left\{ \frac{\vec{s}'}{v'} - \frac{\vec{s}}{c} \right\}$$
 must be

perpendicular to the boundary.



(Double refraction, construction for
Permissible wave normals)

The permissible wave normals \vec{s}' may be determined as follows; with any point O on Σ as origin we plot vectors

(P.T.O)

(from Page-1)

\Rightarrow in all directions \vec{s}' , each of length $\frac{1}{\sqrt{v}}$ where v is the phase velocity corresponding to each \vec{s}' in accordance with Fresnel's equation. The locus of the end points is a two sheeted surface which differs from the normal surface in that each radius vector is of length $\frac{1}{\sqrt{v}}$ instead of v . We call this surface the inverse surface of wave normals. It is dual of the ray surface and, therefore like the ray surface itself, is of the fourth degree. Since the required vector \vec{s}' must be such that $\vec{s}' - \vec{s}$ is perpendicular to Σ , its end point Q' must be on the normal to Σ through the end point P of the vector \vec{s} . In general, the normal to normal to Σ cuts the inverse surface in four points, two of which lie in the same side of the boundary as the crystal.

Hence there are two such points Q' and Q'' in figure-(1), and therefore two possible wave normal directions, so that in general each incident wave will give rise to two refracted waves, to each of these waves there corresponds a ray direction and a ray direction and a ray velocity describing the propagation of the energy within the crystals. This is the phenomenon of double refraction. It is illustrated by the well known effect that two images are observed when a small object is viewed through a slab of Calcite.

$$\text{We have, } \frac{\sin \theta_i}{\sin \theta_t} = \frac{c}{v} = \frac{\sin \theta_i}{\sin \theta''_t} = \frac{c}{v''} \rightarrow \textcircled{2}$$

Hence each of the transmitted wave is seen to obey the same law of refraction.