

Paper-3rd : Total internal reflection of E.M.waves.

If a linearly Polarised wave is reflected from the boundary at an angle greater than the critical angle, the reflected wave will be elliptically Polarised.

Let us suppose that a plane E.M.wave is incident on the interface from a denser medium to rarer medium. The angle of incidence θ_i is in the denser medium and angle of refraction θ_T is in the rarer medium as shown in fig-(1). The word internal implies that the incident and reflected waves are in denser medium than the refracted wave, this means that $n_1 > n_2$.

From Snell's law we know

$$\text{that } \sin \theta_i = \frac{n_2}{n_1} \sin \theta_T \rightarrow (1)$$

$$\therefore \theta_i < \theta_T [\because \frac{n_2}{n_1} < 1]$$

So, for a particular value of angle of incidence θ_i will become 90° .

If θ_i is increased gradually.

The value of angle of incidence θ_i for which $\theta_T = 90^\circ$ is called critical angle and is denoted θ_c so far,

$$\sin \theta_c = \frac{n_2}{n_1} \sin 90^\circ = \frac{n_2}{n_1} \rightarrow (2)$$

In the case of refracted wave is propagated parallel to the boundary surface. If we further increase θ_i the refracted wave will turn back in its previous medium. In this situation from eqn. (1) in the light of eqn. (2), we get.

$$\sin \theta_T = \frac{\sin \theta_i}{\left(\frac{n_2}{n_1}\right)} = \frac{\sin \theta_i}{\sin \theta_c} > 1 (\because \theta_i > \theta_c) \rightarrow (3)$$

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Equation (3) shows the imaginary character of θ_T . To find the meaning of imaginary θ_T , let us consider the transmitted E.M wave,

$$\text{Eqn } \rightarrow \rightarrow -i(\omega t - k_z z) \\ \text{or, } \vec{E}_T = \vec{E}_0 e^{-i(\omega t - k_z z)}$$

$$\text{or, } \vec{E}_T = \vec{E}_0 i[\omega t - k_T (x \sin \theta_T + z \cos \theta_T)] \rightarrow (4)$$

But from equation (3), Equation (4) reduces to

$$\vec{E}_T = \left\{ \vec{E}_0 e^{-k_T z} \right\} e^{-i[\omega t - k_T (\sin \theta_i / \sin \theta_c) z]} \rightarrow (5)$$

Equation (5) shows that for $\theta_i > \theta_c$ the transmitted wave is propagated only parallel to the surface and is attenuated exponentially beyond the interface.

Now from Fresnel's equation, we have,

$$\left(\frac{E_R}{E_T} \right)_\perp = \frac{\cos \theta_i - \frac{n_2}{n_1} \cos \theta_T}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_T} = \frac{a - ib}{a + ib}$$

$$\text{where } a = \cos \theta_i \text{ and } b = \frac{n_2}{n_1} \cos \theta_T$$

Solving these equations, we get, -

$$\left(\frac{E_R}{E_T} \right)_\perp = e^{-\phi_\perp} \quad \text{where } \tan \frac{\phi_\perp}{2} = \frac{b}{a} \rightarrow (6)$$

Similarly, where E is parallel to the plane of incidence

$$\left\{ \frac{E_R}{E_i} \right\}_{||} = \frac{\cos \theta_i - \frac{n_1}{n_2} \cos \theta_T}{\cos \theta_i + \frac{n_1}{n_2} \cos \theta_T} = \frac{a - ib'}{a + ib'}$$

$$\text{where } a = \cos \theta_i \text{ and } b' = \frac{n_1}{n_2} \cos \theta_T$$

$$\therefore (E_R/E_i)_{||} = e^{-\phi_{||}} \quad \text{with } \tan \frac{\phi_{||}}{2} = -\frac{b'}{a} \rightarrow (7)$$

Therefore from eqn. (6) and (7) it is clear that the amplitudes and the intensity of the reflected wave is equal to that of incident wave.