

Notes for

—By.

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B.Sc Part-II

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Paper-3rd. "Pressure of radiation."

The appearance of a field momentum becomes understandable when we consider that it is possible for electromagnetic radiation to convey not only energy but also momentum for source of radiation to an observer and this conveyed momentum is measurable ~~as momentum~~ as radiation pressure on the absorber.

For calculating the radiation pressure,

let us consider a surface element with normal in the positive direction of x-axis. As the term $\oint T_{ik} \cdot d\vec{s}$ represents the force on the boundary.

$$\vec{F}_x = \oint_S T_{xk} \vec{n}_x ds$$

$$\text{or, } \vec{F}_x = \oint_S (T_{xx} + T_{xy} + T_{xz}) \vec{n}_x ds$$

$$\text{or, } \vec{F}_x = \oint_S T_{xx} ds$$

So, radiation pressure which is force per unit ~~area~~ area in a direction opposite to that of \vec{F}_x will be

$$P_{rad} = -T_{xx}$$

$$\text{But } T_x = \left[\epsilon_0 (E_x^2 - \frac{1}{2} E^2) + \mu_0 (H_x^2 - \frac{1}{2} H^2) \right]$$

$$\therefore P_{rad} = - \left[\epsilon_0 (E_x^2 - \frac{1}{2} E^2) + \mu_0 (H_x^2 - \frac{1}{2} H^2) \right] \quad ①$$

Now, as for a light wave \vec{E} and \vec{H} are transverse to the direction of wave propagation (in the x-direction)

$$\therefore E_x = H_x = 0$$

$$\therefore P_{rad} = \left[\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right] = u \rightarrow ② \Rightarrow$$

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→ where u is the energy density.

$$\text{or } P_{\text{rad}} = u = \frac{S}{c} \quad \therefore S = uc \rightarrow ③$$

where S is the gradient energy per unit area falling per second normally on the surface. This equation holds good only if the surface absorbs all the radiation incident on it. However, if the surface reflects all the incident radiation, then

$$P_{\text{rad}} = 2u = \frac{2S}{c} \rightarrow ④$$

The wave is reflected with momentum equal to its incident momentum but in opposite direction. Hence the factor 2 has been introduced in equation ④.

For isotropically incident radiations (uniform from all directions) we have for the average

$$\bar{E_x^2} = \frac{E^2}{3} \text{ and } \bar{H_x^2} = \frac{H^2}{3}$$

∴ from equation ④ we have -

$$P_{\text{rad}} = - \left[\epsilon_0 \left(\frac{\bar{E^2}}{3} - \frac{\bar{E^2}}{2} \right) + \mu_0 \left(\frac{\bar{H^2}}{3} - \frac{\bar{H^2}}{2} \right) \right]$$

$$\text{or, } P_{\text{rad}} = \frac{1}{6} [\epsilon_0 \bar{E^2} + \mu_0 \bar{H^2}] = \frac{u}{3} \text{ (from eq. 4)} \rightarrow ⑤$$

This equation holds good for all surfaces (either absorber or reflector) on account of perfect symmetry.

The radiation pressure of an electromagnetic wave has been observed experimentally and has been found to agree with theory.