

Notes for B.Sc Part - III
Paper - 7th.

Free electron theory of metals.

The valence electrons are loosely bound to their individual atoms in the metals. They move randomly in different directions and are called free electrons. The binding forces on these electrons are very small and they behave like molecules of gas and they form a free electron cloud or a gas in metals. It is matter to notice that except free electrons all the electrons are tightly bound to the nuclei.

Let us consider copper atom. It has 29 electrons. The 28 electrons of it are tightly bound to the nuclei and one valence electron is free.

In a metallic crystal the valence electrons of the constituent atoms are free to move throughout the crystal lattice. The positive ions of the crystal lattice produce an attractive potential for the electrons, which is everywhere the same within the crystal and zero outside.

The valence electrons, therefore may be considered to occupy energy levels within a potential well. At absolute zero of temperature, the maximum kinetic energy that a free electron can have is called the Fermi-energy, E_F .

Let us assume the distribution of electrons among the energy levels in an energy band. According to F.D. Statistics the no of electrons having their energies lying between E and $E+dE$ is

$$N(E)dE = \frac{Z(E)dE}{e^{E/kT}} \rightarrow ①$$

(P.T.O)

(from page - 1)

→ where $Z(E)dE$ = the no of quantum states of electrons having their energies lying between E and $E+dE$ is

$$N(E)dE = \frac{Z(E)dE}{\text{as above}} \cdot \frac{d\tau}{e^{E/kT}}$$

The number of quantum states within three dimensional momentum range $d\mathbf{p}$ and three dimensional space range $d\tau$ may be written as

The no of quantum states within three dimensional

$$Z(\tau, \mathbf{p})d\tau d\mathbf{p} = \frac{d\mathbf{p}_x d\mathbf{p}_y d\mathbf{p}_z d\tau}{h^3}$$

where τ = space volume, \mathbf{p} = momentum.

for entire space volume V , the number of quantum states between \mathbf{p} and $\mathbf{p}+d\mathbf{p}$ is

$$Z(\mathbf{p})d\mathbf{p} = \frac{2V d\mathbf{p}_x d\mathbf{p}_y d\mathbf{p}_z}{h^3}$$

The factor '2' has been introduced because the electron has two degeneracy on account of spin.

Now from the relation,

$$\sqrt{x^2 + y^2 + z^2} = V \quad \text{we get,}$$

$$p_x^2 + p_y^2 + p_z^2 = p^2$$

which represents sphere of radius p . Hence

$$d\mathbf{p}_x d\mathbf{p}_y d\mathbf{p}_z = 4\pi p^2 dp$$

$$\therefore Z(p)dp = \frac{4\pi p^2 dp}{h^3} \times 2V \rightarrow ②$$

If E and $E+dE$ be the energy corresponding to the momentum p and $p+dp$ then $Z(E)dE$ will represent the number of quantum states within the energy range E and $E+dE$.

