Hyperbolic Function

Hyperbolic functions are analogs of circular fns. Hyperbolic fns. are wholly real i.e. i-less and are denoted by $\sinh\theta$, $\cosh\theta$, $\tanh\theta$, $\coth\theta$, $\operatorname{sech}\theta$, $\operatorname{cosech}\theta$.

Definition:- If θ be real or complex the quantity $\frac{e^{\theta}+e^{-\theta}}{2}$, $\frac{e^{\theta}-e^{-\theta}}{2}$ are called hyperbolic cosine of θ and hyperbolic sine of θ respectively.

The other hyperbolic fns. are defined from hyperbolic sine or cosine just as the ordinary circular fns are obtained. i.e.

We have $\sinh\theta = \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)$, $\cosh\theta \left(\frac{e^{\theta}+e^{-\theta}}{2}\right)$

$\tanh\theta = \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)$, $\coth\theta = \left(\frac{e^{\theta}+e^{-\theta}}{e^{\theta}-e^{-\theta}}\right)$

$\operatorname{sech}\theta = \left(\frac{2}{e^{\theta}+e^{-\theta}}\right)$, $\operatorname{cosech}\theta = \left(\frac{2}{e^{\theta}-e^{-\theta}}\right)$

- Relation between Circular fn and hyperbolic fns. :—

For all values of x, we have

$$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$$

Putting $x = ix$, we get

$$\sin(ix) = \frac{1}{2i}\left(e^{i^2x} - e^{-i^2x}\right) = \frac{1}{2i}\left(e^{-x} - e^{x}\right) = -\frac{1}{2i}(e^{x} - e^{-x})$$

$$= -\frac{i}{2i^2}(e^{x} - e^{-x}) = + \frac{e^{x}-e^{-x}}{2}$$

$$\therefore \boxed{\sin(ix) = i\sinh x}$$

and we can find easily that $\boxed{\sinh\theta = -\sin ix}$.

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Thus we can find

$$\sin i\theta = i\sinh\theta$$
$$\cos i\theta = \cosh\theta$$
$$\tan i\theta = i\tanh\theta$$

And also
$$\sinh\theta = -i\sin i\theta$$
$$\cosh\theta = \cos i\theta$$
$$\tanh\theta = -i\tan i\theta$$

- Prove that $\cosh^2\theta - \sinh^2\theta = 1$.

Proof:- L.H.S. $= \cosh^2\theta - \sinh^2\theta$

$$= \left(\frac{e^{\theta}+e^{-\theta}}{2}\right)^2 - \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)^2$$

$$= \frac{1}{4}\left(e^{2\theta}+e^{-2\theta}+2\right) - \frac{1}{4}\left(e^{2\theta}+e^{-2\theta}-2\right)$$

$$= \frac{1}{4}\left[e^{2\theta}+e^{-2\theta}+2-e^{2\theta}-e^{-2\theta}+2\right] = 1 = \text{R.H.S.}$$

Proved

- Periods of Hyperbolic functions.

$\because$ we can easily show that

(i) $\sinh(\theta+n\pi i) = -\sinh\theta$

(ii) $\cosh(\theta+n\pi i) = -\cosh\theta$

(iii) $\tanh(\theta+in\pi) = \tanh\theta$

Thus the periods of $\sinh\theta$ and $\cosh\theta$ is $2\pi i$ and the period of $\tanh\theta$ is $i\pi$.

Hence Hyperbolic fns. are periodic fns. with imaginary period.

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