

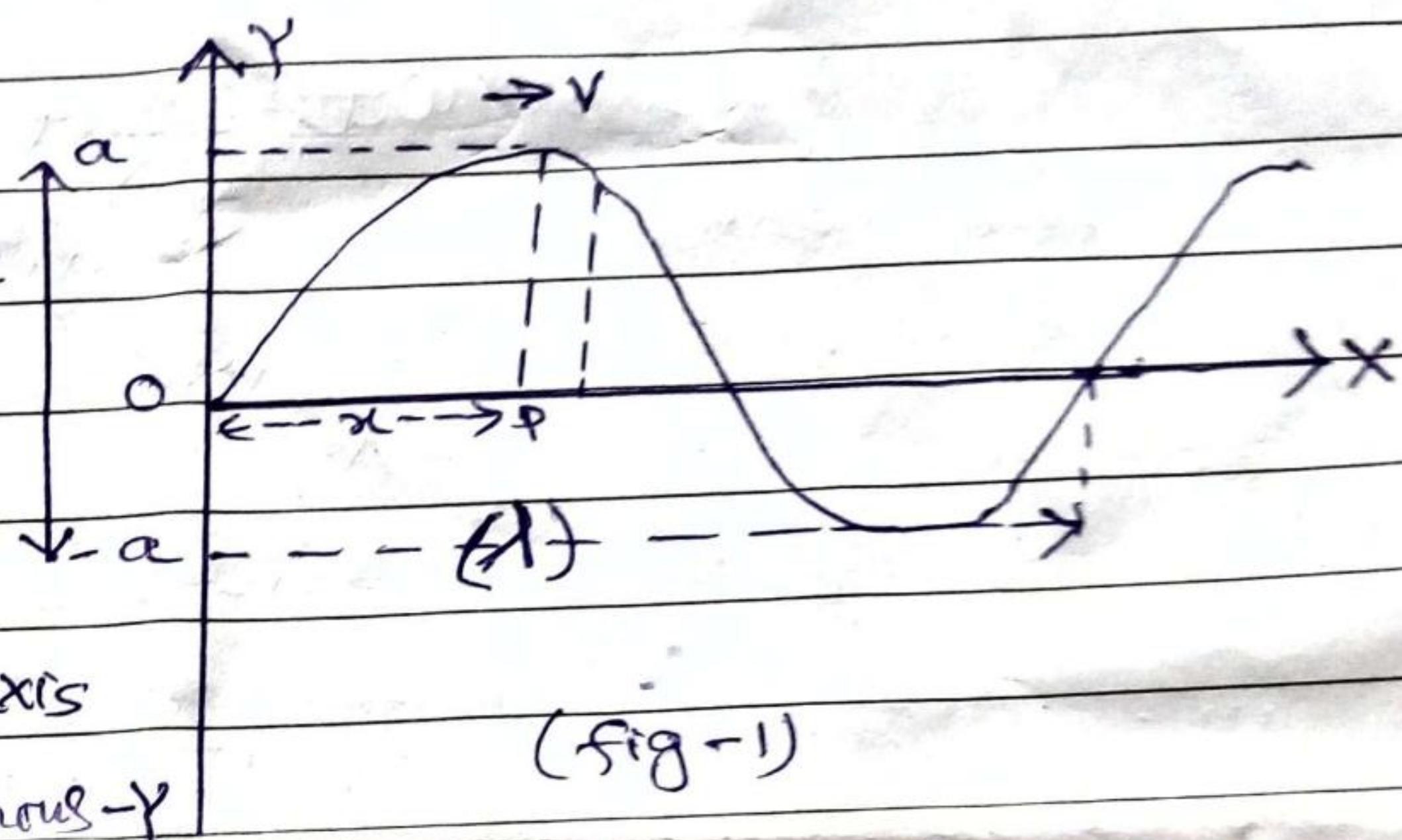
Notes for:

B.S.C.-I

Paper-Ist: Equation of Progressive wave.

A plane Progressive wave harmonic wave is the simplest wave in which the particles of the particles of the medium perform simple harmonic motion along the direction of propagation.

Let us suppose that the source of simple harmonic disturbance is situated at the origin O, and the wave travels in the positive direction of X-axis in a continuous homogeneous



(fig-1)

medium. As the wave proceeds onward each successive particles of the medium is set in the simple harmonic oscillation. Fig(1) shows the displacements (y) of the particles against their positions (x) at a particular instant (t). If we measure time from the instant when the particle at O is passing through its mean position, then the displacement of this particle at any instant (t) along Y-axis is given by -

$$y = a \sin \omega t \rightarrow ①$$

where a is the amplitude of the particles at the periodic time T which is the time taken by the wave to cover a distance λ (wavelength). The displacement at the same instant of a particle P at a distance xe from O towards the right can be written as - $y = a \sin(\omega t - \phi) \rightarrow ②$

(P.T.O)

In page 1, where ϕ is the Phase lag since it is evident that the particle situated towards the right of O must always be behind the particle at O in phase if the wave is travelling towards the right.

We know that for a path difference λ , phase change is $\phi \therefore \frac{2\pi}{\lambda} = \frac{\phi}{\lambda} \Rightarrow \phi = 2\pi \frac{x}{\lambda}$

$$\text{Again, } \omega t = \frac{2\pi}{\lambda} \cdot t = \frac{2\pi v}{\lambda} \quad [\because v = \frac{\lambda}{T}]$$

∴ From equation (2) we get,

$$y = a \sin \left\{ \frac{2\pi}{\lambda} vt - \frac{2\pi}{\lambda} x \right\}$$

$$\text{or } y = a \sin \frac{2\pi}{\lambda} (vt - x) \rightarrow (3)$$

This equation gives us the complete form of a plane progressive wave of amplitude (a) with a velocity of propagation v in the positive direction of x -axis. When x is measured opposite to the direction of propagation the above equation becomes,

$$y = a \sin \frac{2\pi}{\lambda} (vt + x) \rightarrow (4)$$

The quantity $\frac{2\pi}{\lambda} = 2\pi \frac{v}{\lambda} = k$ is called Propagation constant, while $\frac{1}{\lambda} = \frac{v}{\lambda}$ is called the wave number. Hence eqn. (3) reduces to

$$y = a \sin (wt - kx) \rightarrow (5)$$

at $t=0, x=0$, and $y=0$

All the above equations from eqn. (1) to eqn. (5) represent the equation of a plane Progressive wave in different forms.

Properties: — For a fixed value of x , the displacement y is a simple harmonic oscillation in time shows with the same amplitude and period.

