

Notes for:

B.Sc. Part-III
Paper - 6th

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Application of Bose-Einstein Statistics"

An application of Bose-Einstein statistics to photons can be described. Radiation from a black-body at absolute temperature T , and in thermal equilibrium is supposed to consist of light quanta or photons of energy content $h\nu$, moving in all possible directions with speed of light c and, therefore possesses the momentum $h\nu/c$.

Some of the important properties of the photons are-

- (i) Photons are particles of zero rest masses.
- (ii) Photons are indistinguishable from one another and their number in a system is not necessarily constant, the emission of several photons of frequencies ν_1, ν_2, \dots provided the total energy of the system is constant. i.e.,

$$h\nu = h\nu_1 + h\nu_2 + \dots$$

Thus in this case, we have, $\sum_i \delta n_i \neq 0 \rightarrow \textcircled{1}$

Hence, we must drop the auxiliary condition $\sum_i \delta n_i = 0$ and therefore, undetermined multiplier α is equal to zero.

- (iii) The photons are Bose particles with spin 1, having two modes of propagation. According to quantum idea each allowed eigen state of a quantum mechanical system has a volume h^3 in the phase space. The volume of each allowed eigen state can be written as -

$$d\tau = dx dy dz dp_x dp_y dp_z = h^3 \rightarrow \textcircled{2}$$

According to Heisenberg's uncertainty relation an element of volume in the momentum space can be written as

$$\sigma_p = \frac{h^3}{V} \rightarrow \textcircled{3}$$

As σ_p denotes the size of an elementary cell in the

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→ momentum space, only a single value of momentum can be recognized within a cell. Therefore this represents an eigen state.

The total number of eigen states between momenta p and $p+dp$ is given by - $g(p)dp = \frac{4\pi p^2 dp}{h^3/v} = \frac{4\pi p^2 v}{h^3} dp \rightarrow \textcircled{4}$

For a photon, $p = \frac{h\nu}{c}$ or, $dp = \frac{h d\nu}{c}$

Substituting these values in equation $\textcircled{4}$ the total number of eigen states between frequencies ν and $\nu+d\nu$ is given by -

$$g(\nu) d\nu = 4\pi V \cdot \frac{\nu^2}{c^3} d\nu \rightarrow \textcircled{5}$$

Taking in to account the doubling of the states due to polarisation of the photons (i.e. two modes of propagation for each photon) the total number of eigen states available for the photons in the frequency range ν and $\nu+d\nu$ is given by

$$g(\nu) d\nu = 8\pi V \cdot \frac{\nu^2}{c^3} d\nu \rightarrow \textcircled{6}$$

in this case $\alpha = 0$, $\beta = \frac{1}{KT}$ and $\epsilon = h\nu$ (the energy of photon)

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there, using $\textcircled{6}$ above equation may be written as

$$dn = 8\pi V \cdot \frac{\nu^2}{c^3} \cdot \frac{d\nu}{e^{h\nu/KT} - 1}$$

$$\text{i.e. } \frac{dn}{V} = \frac{8\pi \nu^2}{c^3} \cdot \frac{d\nu}{e^{h\nu/KT} - 1} \rightarrow \textcircled{7}$$

This is well known Planks law of radiation in terms of frequency. This equation represents the number of photons per unit volume lying in the frequency range ν and $\nu+d\nu$.

Planks law of radiation in terms frequency can also be written as -

$$U_\nu d\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{d\nu}{e^{h\nu/KT} - 1} \rightarrow \textcircled{8}$$

This is Planks law of radiation in terms of frequency.