

Notes for.

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B.Sc Part-II  
Paper - IIIrd

### "Theory of Interference"

The Superposition of two waves of light is called interference.

Let us consider the superposition of two waves of same frequency  $\frac{\omega}{2\pi}$  but differing in phase. If  $a_1$  and  $a_2$  are the amplitudes of the two waves, the displacement due to one wave at any instant  $t$  is represented as -

$$y_1 = a_1 \sin \omega t \quad \rightarrow ①$$

This displacement due to another wave at the same instant is represented as -  $y_2 = a_2 \sin(\omega t + \delta) \rightarrow ②$

where  $\delta$  is the phase difference between the two waves at instant  $t$ .

Now according to Young's Principle of Superposition whenever a particle is disturbed simultaneously by two or more waves, the resultant displacement at any instant will be the sum of the displacement due to each wave separately. i.e. the resultant displacement  $y$  is given by

$$y = y_1 + y_2 \rightarrow ③$$

Substituting values of  $y_1$  and  $y_2$  from eqn. ① and ② in eqn. ③ we get -

$$\begin{aligned} y &= a_1 \sin \omega t + a_2 \sin(\omega t + \delta) \\ &= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta \end{aligned}$$

$$\therefore y = (a_1 + a_2 \cos \delta) \sin \omega t + (a_2 \sin \delta) \cos \omega t \rightarrow ④$$

$$\text{Let us take, } a_1 + a_2 \cos \delta = A \cos \phi \rightarrow ⑤$$

$$a_2 \sin \delta = A \sin \phi \rightarrow ⑥$$

where  $A$  and  $\phi$  are new constants.

Squaring eqn. ⑤ and ⑥ and then adding, we get -

$$(a_1 + a_2 \cos \delta)^2 + (a_2 \sin \delta)^2 = A^2$$

$$\text{or, } A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$

$$\text{or } A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta \rightarrow ⑦$$

$$\Rightarrow A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta} \rightarrow$$

P.T.O.

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→ Dividing equation ⑥ by ⑤ we get -

$$\tan \phi = \frac{a_2 \sin \delta}{a_1 + a_2 \cos \delta} \rightarrow ⑧$$

Using equations ⑤ and ⑥) equation ④ gives -

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

$$\therefore y = A \sin(\omega t + \phi) \rightarrow ⑨$$

obviously, the resultant disturbance has amplitude given by equation ⑦.

The intensity at any point is given by the squares of the amplitude, in arbitrary units

$$\text{ie } I = A^2$$

$$\text{or, } I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta \rightarrow ⑩$$

$$\text{or } I = a_1^2 + a_2^2 + 2a_1 a_2 (2 \cos^2 \frac{\delta}{2} - 1)$$

$$\text{or } I = a_1^2 + a_2^2 - 2a_1 a_2 + 4a_1 a_2 \cos^2 \frac{\delta}{2}$$

$$\text{or } I = (a_1 - a_2)^2 + 4a_1 a_2 \cos^2 \frac{\delta}{2} \rightarrow ⑪$$

As  $a_1$  and  $a_2$  are constants, the intensity  $I$  will vary from point to point in the region of interference of the two waves according to the variation of  $\cos^2 \frac{\delta}{2}$  form.

It is obvious that  $a_1^2$  and  $a_2^2$  are the intensities of the two interfering waves and the resultant intensity at any point is not just the sum of the intensities due to separate waves are the  $(a_1^2 + a_2^2)$ .

