

Notes for:

B.Sc. Part III

Page No - VI Th.

### "Fermi-Dirac distribution"

Let us consider a system having  $n$  indistinguishable particles. Let these particles be divided into quantum groups -  $n_1, n_2, \dots, n_i$  whose constant energies are  $E_1, E_2, \dots, E_i$  respectively. Let  $g_i$  denote the sublevels of  $i$ th level.

In Fermi-Dirac statistics the conditions are —

- (i) The particles are indistinguishable from each other so that, there is no distinction between the different ways in which  $n_i$  particles are chosen.
- (ii) The particles obey Pauli's exclusion principle according to which each sublevel or cell may contain 0 to 1 particle. Then obviously,  $g_i$  must be greater than or equal to  $n_i$ .
- (iii) The sum of energies of all the particles in the different quantum groups taken together constitutes the total energy of the system.

Now the distribution of  $n_i$  particles in  $g_i$  states can be done as following —

Since due to Pauli's principle no cell can occupy more than one particle, therefore, among  $g_i$  cells only  $n_i$  cells are occupied by one particle each and the remaining  $(g_i - n_i)$  cells are empty.

Hence the number of distinguishable arrangements of  $n_i$  particles in  $g_i$  cells is

$$\frac{g_i!}{n_i!(g_i - n_i)!} \rightarrow ①$$

Similar expressions can also be found for various other quantum states. The total number of eigen states for the whole system is given by

$$G = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!} \rightarrow ②$$

The Probability  $W$  of the system is proportional to the total number of eigen states i.e.

$$W = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!} \times \text{constant} \rightarrow ③$$

( P.T.O )

→ (from Page-1)  
 The Fermi-Dirac distribution law can now be obtained by determining the most probable distribution.

Taking log of equation (3) we get -

$$\log W = \sum [ \log g_i! - \log n_i! - \log (g_i - n_i)! ] + \text{constant} \quad (4)$$

As  $n_i$  and  $g_i$  are large numbers, therefore, using Sterling approximation, equation (4) reduces to -

$$\log W = \sum [ (n_i - g_i) \log (g_i - n_i) + g_i \log g_i - n_i \log n_i ] + \text{constant} \quad (5)$$

remembering that  $g_i$  is not subject to variation and  $n_i$  varies continuously. The differentiation of above eqn (5) gives

$$\delta \log W = - \sum_i \left\{ \log \frac{n_i}{g_i - n_i} \right\} \delta n_i \quad (6)$$

for most probable distribution  $W = W_{\max}$  so  $\delta (\log W_{\max}) = 0$

$$\text{or } \sum_i \left\{ \log \frac{n_i}{g_i - n_i} \right\} \delta n_i = 0 \quad (7)$$

The two subsidiary conditions are -

(i) The total number of particles of the system is constant.

$$\text{ie } n = \sum_i n_i = \text{constant} \text{ or } \delta n = \sum_i \delta n_i = 0 \quad (8)$$

(ii) Total energy of the system is constant.

$$\text{ie } E = \sum_i n_i \epsilon_i = \text{constant} \text{ or } \delta E = \sum_i \epsilon_i \delta n_i = 0 \quad (9)$$

Now to apply the Lagrangian method of determined multipliers, we multiply eqn (8) by  $\alpha$  and eqn (9) by  $\beta$  and add the resulting expressions to eqn (7), we get -

$$\sum_i \left[ \log \left\{ \frac{n_i}{g_i - n_i} \right\} + \alpha + \beta \epsilon_i \right] \delta n_i = 0 \quad (10)$$

As the variations  $\delta n_i$  are independent of each other, we get -

$$n_i = \frac{g_i}{e^\alpha + \beta \epsilon_i + 1} \quad (11)$$

This equation (11) is known as Fermi-Dirac distribution law.

x ← → x