

B.Sc III Paper - 7th Dr. Bhawat Singh, Deptt of Physics
 Sher Singh College - Sector 1 (Page-1)
 Q:- Discuss the Phenomenon of ionospheric reflection of electromagnetic waves
 On the Principle of wave propagation in isotropic plasma.

Ans:- Ionospheric reflection of e.m.waves in isotropic Plasma: — It has been found that in the upper atmosphere of the earth the constituent rarefied gases are in a state of ionisation, mainly as a result of ultraviolet radiation from the Sun. This ionised atmosphere, extending upwards from heights of about 50 km, is known as ionosphere. The presence of electrons or ions in an ionised region is to reduce the value of the permittivity below that of free space.

Let us suppose that there is no collision between electrons. The ionised region may then be assumed to have NO free electrons, each of charge e and mass m , per cubic meter, if the region is traversed by an electromagnetic wave which has an electric field \vec{E} and magnetic field \vec{H} , then Maxwell's field equation are

$$\left. \begin{aligned} \operatorname{div} \vec{B} &= 0 \\ \operatorname{div} \vec{D} &= 0 \\ \operatorname{curl} \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \text{and } \operatorname{curl} \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned} \right\} \text{with } \left. \begin{aligned} \vec{J} &= \sigma \vec{E} \\ \vec{B} &= \mu \vec{H} \\ \vec{D} &= \epsilon \vec{E} \end{aligned} \right\} \rightarrow \textcircled{1}$$

where \vec{D} = electric displacement, \vec{B} = magnetic induction \vec{J} current density ϵ = Permittivity of the medium and μ = Permeability of the medium.

In the case of ionised medium $\rho = 0$, $\mu = \mu_0$ and $\epsilon = \epsilon_0$ so, eqn. ① becomes —

$$\left. \begin{aligned} \operatorname{div} \vec{E} &= 0 & \text{--- (a)} \\ \operatorname{div} \vec{H} &= 0 & \text{--- (b)} \\ \operatorname{curl} \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} & \text{--- (c)} \\ \operatorname{curl} \vec{H} &= \sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} & \text{--- (d)} \end{aligned} \right\} \rightarrow \textcircled{2}$$

$$\begin{aligned} \text{Taking curl of eqn. (2c) we get} & \quad \operatorname{curl} \operatorname{curl} \vec{E} = -\mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} (\operatorname{curl} \vec{H}) \\ & = -\mu_0 \frac{\partial}{\partial t} \left\{ \sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right\} \text{ (from (d))} \\ & = -\mu_0 \sigma \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \rightarrow \textcircled{3} \end{aligned}$$

$$\begin{aligned} \therefore \operatorname{curl} \operatorname{curl} \vec{A} &= \operatorname{grad} \operatorname{div} \vec{A} - \nabla^2 \vec{A} \rightarrow \textcircled{4} \\ \text{and } \operatorname{div} \vec{E} &= 0 \quad (\text{from eqn. (2a)}) \rightarrow \\ & \quad (\vec{D} = 0) \end{aligned}$$

$$\therefore \text{curl curl } \vec{E} = \text{grad}(\text{div} \vec{E}) - \vec{\nabla}^2 \vec{E}$$

$$= -\vec{\nabla}^2 \vec{E} \rightarrow \textcircled{5}$$

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From eqn. (3) and (5) we get after solving - $\vec{\nabla}^2 \vec{E} - \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \rightarrow \textcircled{6}$

Again taking curl of eqn. (2d) we get $\text{curl curl} \vec{H} = \sigma \text{curl} \vec{E} + \epsilon_0 \frac{\partial}{\partial t} (\text{curl} \vec{E})$

Substituting the value of $\text{curl} \vec{E}$ from eqn. (2c) we have -

$$\text{curl curl} \vec{H} = -\sigma \mu_0 \frac{\partial \vec{H}}{\partial t} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} \rightarrow \textcircled{7}$$

$$\text{But } \text{curl curl} \vec{H} = \text{grad}(\text{div} \vec{H}) - \vec{\nabla}^2 \vec{H}$$

$$= -\vec{\nabla}^2 \vec{H} \text{ from eqn. } \textcircled{2b}$$

$$\therefore \text{Equation } \textcircled{7} \text{ reduces to } \vec{\nabla}^2 \vec{H} - \sigma \mu_0 \frac{\partial \vec{H}}{\partial t} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \rightarrow \textcircled{8}$$

Eqs. (6) and (8) representing wave eqns. governing electromagnetic fields \vec{E} and \vec{H} in ionized medium. These equations are vector equations of similar form, hence each of six components of \vec{E} and \vec{H} separately satisfies the same scalar wave equations of the form.

$$\vec{\nabla}^2 \psi - \mu_0 \sigma \frac{\partial \psi}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \psi}{\partial t^2} = 0 \rightarrow \textcircled{9}$$

where ψ is a scalar and can stand for any one of six components of \vec{E} and \vec{H} .

The Plane wave Solutions of eqns. (6), (8) and (9) can be taken in the form

$$\vec{E} = \vec{E}_0 e^{i \vec{k} \cdot \vec{r} - i \omega t} \rightarrow \textcircled{10}$$

$$\vec{H} = \vec{H}_0 e^{i \vec{k} \cdot \vec{r} - i \omega t} \rightarrow \textcircled{11}$$

$$\text{and } \psi = \psi_0 e^{i \vec{k} \cdot \vec{r} - i \omega t} \rightarrow \textcircled{12}$$

where \vec{E}_0 , \vec{H}_0 and ψ_0 are complex amplitudes which are constant in space and time while $\vec{k} = \vec{k} \vec{n} = \frac{2\pi}{\lambda} \vec{n} = \frac{\omega}{v} \vec{n}$ where \vec{n} is a unit vector along \vec{k} and v is the phase velocity of the wave.

From eqn. (12) we have $\frac{\partial \psi}{\partial t} = -i \omega \psi$ and $\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$.

Substituting these values in eqn. (9) and noting that $\vec{\nabla}^2 \psi = -k^2 \psi$ we get - $-k^2 \psi + 2\omega \mu_0 \sigma \psi + \mu_0 \epsilon_0 \omega^2 \psi = 0$

or ~~$\vec{E}_0 \text{ and } \vec{H}_0$~~ $(k^2 - i\omega \mu_0 \sigma - \mu_0 \epsilon_0 \omega^2) \psi = 0$

as ψ is an arbitrary component of field vector hence $\psi \neq 0$

$$k^2 - i\omega \mu_0 \sigma - \mu_0 \epsilon_0 \omega^2 = 0 \rightarrow (P.T.O)$$

~~From page - 2~~ or $\kappa^2 = i\omega \mu_0 \sigma + \kappa e \omega^2$

or $\kappa^2 = \mu_0 \epsilon_0 \omega^2 \left(1 + \frac{i\sigma}{\omega \epsilon_0}\right) \rightarrow 13$

We have assumed that there is no collision between electrons hence there is no loss of energy. In such case the pressure of ionised gases at ionosphere is quite low and the conductivity σ becomes purely imaginary. The conductivity (σ) is then given by

$$\sigma = \frac{i N_0 e^2}{m \omega}$$

∴ Substituting this value of σ in eqn 13 and then

Solving we get - $\kappa^2 = \frac{\omega^2}{c^2} \left[1 - \frac{\omega^2 p}{\omega^2}\right] \rightarrow 14$

$$\text{where } \omega^2 p = N_0 e^2 / m = \epsilon_0$$

ω_p is known as plasma angular frequency

If n is the refractive index of Plasma, then

$$n = \frac{c}{v} \text{ i.e., } v = \frac{c}{n}$$

$$\therefore \kappa = \frac{\omega}{v} = \frac{n \omega}{c}$$

Squaring on both sides we get $\kappa^2 = \frac{n^2 \omega^2}{c^2}$

Substituting this value of κ^2 in eqn 14, we get -

$$n^2 = \left[1 - \frac{\omega^2 p}{\omega^2}\right] \rightarrow 15$$

For high frequency region $\omega > \omega_p$, the refractive index

$$n = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \text{ is real and therefore waves propagate freely}$$

For low frequency region $\omega < \omega_p$ the refractive index n is purely imaginary and then $\kappa = i n \omega \rightarrow 16$

Hence κ (10) and (11) for low frequency region may be written as

$$\vec{E} = \vec{E}_0 e^{i\kappa(\vec{n} \cdot \vec{r})} e^{-i\omega t}$$

$$\text{or } \vec{E} = \vec{e} \left(\frac{\omega n}{c}\right) (\vec{n} \cdot \vec{r}) e^{-i\omega t} \rightarrow 17$$

$$\text{and } \vec{H} = \vec{H}_0 \vec{e} \left(\frac{\omega n}{c}\right) (\vec{n} \cdot \vec{r}) e^{-i\omega t} \rightarrow 18$$

These equations indicate that within the plasma the electromagnetic field vectors \vec{E} and \vec{H} will fall off exponentially with distance from the surface.

