

"Bernoulli's Theorem"

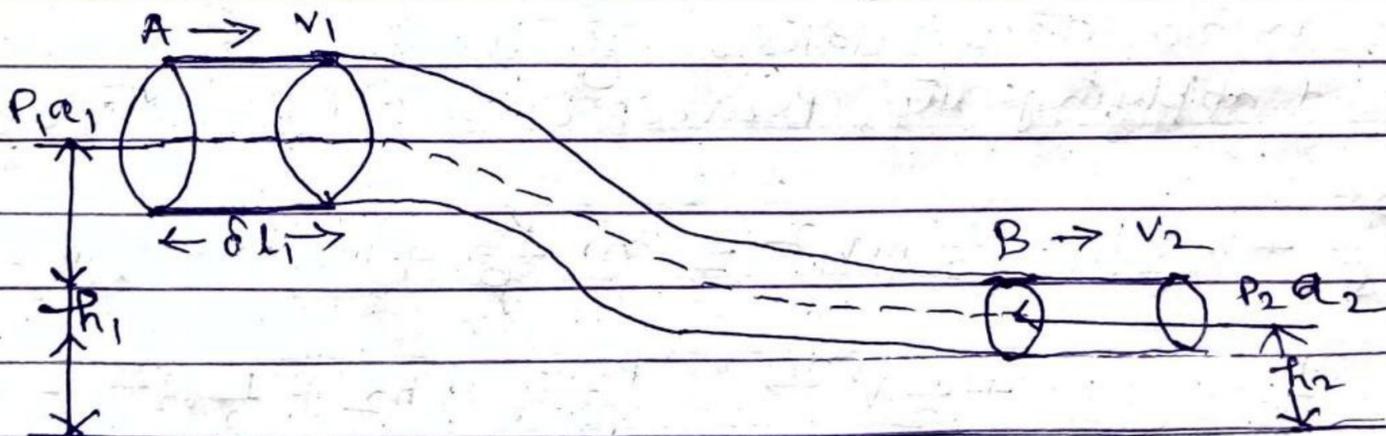
Statement: - "When a non-viscous and incompressible liquid flows steadily in stream lines, the sum of pressure energy, kinetic energy, and potential energy per unit mass at any point in a stream line remains constant."

Mathematically, the theorem can be represented as -

$$\frac{P}{\rho} + hg + \frac{1}{2}v^2 = \text{constant} \rightarrow (1)$$

where  $\frac{P}{\rho}$  is the pressure head,  $hg$  or the potential energy per unit mass due to pressure,  $hg$  is the potential energy per unit mass due to gravity and  $\frac{1}{2}v^2$  is the kinetic energy per unit mass.

Proof: - Let us consider a stream line liquid flow through pipe AB as shown in the following figure.



The energy of a liquid in motion at any point consists of the following three forms -

- (I) Potential energy (II) Kinetic energy (III) Pressure energy.

(I) Potential energy: - The potential energy of a liquid of mass  $m$  at height  $h$  above the ground level =  $mgh$

$\therefore$  Potential energy per unit mass =  $gh$

and P.E per unit volume =  $\rho gh$

where  $\rho$  is the density of mass per unit volume.

(II) Kinetic energy: - The kinetic energy of a liquid of  $\rightarrow$   
in (P.T.O)

→ mass  $m$  moving with a velocity  $v$  is given by  $K.E = \frac{1}{2} m v^2$

and  $K.E$  per unit ~~volume~~<sup>mass</sup> ~~where~~  $\rho$  is the density  
 $= \frac{1}{2} v^2$  and  $K.E$  per unit volume  $= \frac{1}{2} \rho v^2$

(iii) Pressure Energy : — Let  $a_1$  be the area of cross section of the tube  
 $v_1$  be the velocity of the liquid,  $P_1$  be the Pressure at A and  $a_2 v_2$  and  
 $P_2$  the corresponding values at B, then force exerted by the liquid  
at A  $= P_1 a_1$ . Distance travelled by liquid in time  $t = v_1 t$

∴ Work done  $= P_1 a_1 v_1 t$  But  $a_1 v_1 t = V$  (Here  $V$  is the volume of the liquid)

∴ Work done  $= P_1 V$  Hence the Pressure energy at A  $= P_1 V$

If  $m$  is the mass of the liquid, then Pressure energy of the liquid  
at A  $= m \frac{P_1}{\rho}$  and Potential energy of the liquid at A  $= m g h_1$ ,  
where  $h_1$  is the height from the ground level.

Kinetic energy of the liquid at A  $= \frac{1}{2} m v_1^2$

∴ Total energy at A  $= m \frac{P_1}{\rho} + m g h_1 + \frac{1}{2} m v_1^2 \rightarrow (1)$

Similarly, Total energy at B  $= m \frac{P_2}{\rho} + m g h_2 + \frac{1}{2} m v_2^2 \rightarrow (2)$

Since there is no accumulation of liquid anywhere in  
the tube and applying the Principle of conservation of Energy.

$$m \frac{P_1}{\rho} + m g h_1 + \frac{1}{2} m v_1^2 = m \frac{P_2}{\rho} + m g h_2 + \frac{1}{2} m v_2^2$$

$$\text{or, } \frac{P_1}{\rho} + g h_1 + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + g h_2 + \frac{1}{2} v_2^2 = \text{constant.}$$

$$\text{or } \boxed{\frac{P}{\rho} + g h + \frac{1}{2} v^2 = \text{constant.}} \rightarrow (3)$$

This is Bernoulli's theorem.

Dividing eq. (3) by  $g$ , we get -

$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant} \rightarrow (4)$$

where  $\frac{P}{\rho g}$  is called Pressure head,  $h$ , the elevation head and  $\frac{v^2}{2g}$   
the velocity head.

