10-2-2026

BCA - 1st

Q. Prove by Principle of Mathematical Induction that $1 \times \lfloor 1 + 2 \times \lfloor 2 + 3 \times \lfloor 3 + \dots + n \times \lfloor n = \lfloor (n+1) - 1$ for all natural numbers n.

Proof → Let $P(n)$ be the given statement, i.e.,

$$P(n): 1 \times \lfloor 1 + 2 \times \lfloor 2 + 3 \times \lfloor 3 + \dots + n \times \lfloor n = \lfloor n+1 - 1$$

$\forall$ natural numbers n.

($\forall \rightarrow$ for all)

Note that $P(1)$ is true,

Since $P(1): 1 \times \lfloor 1 = 1 = \lfloor 2 - 1 = 2 - 1 = 1$

Assume that $P(n)$ is true for some natural number k,

that is;

$$P(k): 1 \times \lfloor 1 + 2 \times \lfloor 2 + 3 \times \lfloor 3 + \dots + k \times \lfloor k$$
$$= \lfloor k+1 - 1$$

To prove $P(k+1)$ is true, we have

$$P(k+1): 1 \times \lfloor 1 + 2 \times \lfloor 2 + 3 \times \lfloor 3 + \dots + k \times \lfloor k + (k+1) \times \lfloor k+1$$

$$= \lfloor k+1 - 1 + \lfloor (k+1) \times (k+1)$$

$$= \lfloor k+1 \, (k+1+1) - 1 \quad \text{(Taking common } \lfloor k+1 \text{)}$$

$$= \lfloor k+1 \, (k+2) - 1 = \lfloor k+2 - 1$$

Thus $P(k+1)$ is true, whenever $P(k)$ is true.

Therefore, by the principle of mathematical induction, $P(n)$ is true for all natural numbers n. Proved

Matrices :-

A matrix is an ordered rectangular array of numbers (or functions). For example:-

$$A = \begin{bmatrix} x & 5 & 4 \\ 5 & 4 & x \\ 4 & x & 5 \end{bmatrix}$$

Order of a Matrix — A matrix having m rows and n columns is called a matrix of order $m \times n$ or simply $m \times n$ matrix. In above example, A is a matrix of order 3×3.

In general, an $m \times n$ matrix has the following rectangular array :—

$$A = [a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

where $1 \leq i \leq m$, $1 \leq j \leq n$; and $i, j \in N$.

The element a_{ij} is an element lying in the i th row and j th column and is known as the (i, j) th element of A. The number of elements in an $m \times n$ matrix will be equal to mn.

Types of Matrices :-

(i) Row Matrix — A matrix is said to be row matrix if it has only one row.

(ii) Column Matrix — A matrix is said to be column matrix if it has only one column.

(III) Square Matrix — A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix. Thus, $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as square matrix of order n.

(IV) Diagonal Matrix — A square matrix $B = [b_{ij}]_{n \times n}$ is said to be a diagonal matrix if its all non diagonal elements are zero, that is a matrix $B = [b_{ij}]_{n \times n}$ is said to be a diagonal matrix if $b_{ij} = 0$ when $i \neq j$.

(V) Scalar Matrix — A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$b_{ij} = 0$, when $i \neq j$

$b_{ij} = k$, when $i = j$, for some constant k.

eg:- $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(VI) Identity Matrix — A square matrix in which elements in the diagonal are all 1 and rest are all zeroes is called an identity matrix.

In other words, the square matrix

$A = [a_{ij}]_{n \times n}$ is an identity matrix, if

$a_{ij} = 1$, when $i = j$ and $a_{ij} = 0$, when $i \neq j$.

eg:- $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a 3×3 identity matrix.

(VII) Null Matrix — A matrix is said to be null matrix if all of its elements are zeroes. We denote zero matrix or null matrix by O.

(VIII) Equal Matrix — Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(a) They are of the same order and

(b) Each element of A is equal to the corresponding element of B, that is,

$a_{ij} = b_{ij}$ for all i and j.

Multiplication of Matrices:-

The multiplication of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

Let $A = [a_{ij}]$ be an $m \times n$ matrix

and $B = [b_{jk}]$ be an $n \times p$ matrix

Then the product of the matrices A and B is the matrix C of order $m \times p$.

To get the (i, k)th element c_{ik} of the matrix C, we take the ith row of A and kth column of B, multiply them element wise and take the sum of all these products, that is;

$$c_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + a_{i3} b_{3k} + \cdots + a_{in} b_{nk}.$$

The matrix $C = [c_{ik}]_{m \times p}$ is the product of A and B.

Transpose of a Matrix –

Let A be a $m \times n$ matrix. Then A^t, the transpose of a matrix A, is the matrix obtained by interchanging the rows and columns of A. In other words if $A = [a_{ij}]$, then

$$(A^t)_{ji} = a_{ij}.$$

Consequently A^t is $n \times m$ matrix.

Properties of Transpose Matrix –

(1) $(A^t)^t = A$; (2) $(A \pm B)^t = A^t \pm B^t$ if A and B are $m \times n$ matrices.

(3) $(sA)^t = sA^t$ if s is a scalar;

(4) $(AB)^t = B^t A^t$ if A is $m \times n$ matrix and B is $n \times p$ matrix.

(5) If A is non-singular matrix then A^t is also non-singular matrix and $(A^t)^{-1} = (A^{-1})^t$;

(6) $X^t X = x_1^2 + \cdots + x_n^2$ if $X = [x_1, x_2, \cdots x_n]^t$ is a column vector.