3-2-26 BCA - 1st Sem

Second Principle of Mathematical Induction:-

A set of positive integers which contains the integer 1, and which has the property that if it contains all the positive integers $1, 2, 3, \cdots, k$, then it also contains the integer $k+1$, must be the set of all positive integers.

Proof:-

Let T be a set of integers containing 1 and containing $k+1$ if it contains $1, 2, 3, \cdots, k$.

Let S be the set of all positive integers n such that all the positive integers less than or equal to n are in T.

Then 1 is in S, and by the hypotheses, we see that if k is in S, then k+1 is in S.

Hence, by the principle of mathematical induction, S must be the set of all positive integers, so clearly T is also the set of all positive integers.

Q. Adding Odd Numbers formula:-

$$1+3+5+\cdots+(2n-1) = n^2$$

Proof:- Let us put $n=1$ in both sides of the above formula:-

$$1 = 1^2 \text{ which is true.}$$

Now Assume it is true for $n=k$

$\therefore 1+3+5+\cdots+(2k-1) = k^2$ is true

Now, we have to prove that it is true for "k+1"

$$1+3+5+\cdots+(2k-1)+(2(k+1)-1) = (k+1)^2$$

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We know that-

$1+3+5+\cdots+(2k-1) = k^2$ (According to the assumption above),

So, we can do a replacement for all but the last term :

$k^2 + (2(k+1)-1) = (k+1)^2$

Now expanding all terms we get —

$k^2 + 2k + 2 - 1 = k^2 + 2k + 1$

And simplify :-

$\Rightarrow k^2 + 2k + 1 = k^2 + 2k + 1$ (LHS = RHS)

They are the same ! so it is true.

So, we can say that-

$1+3+5+\cdots+(2(k+1)-1) = (k+1)^2$ is True.

Hence, $1+3+5+\cdots+(2n-1) = n^2$ is True

Q Cubes of Natural Numbers formula :-

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4} n^2 (n+1)^2$

Proof :- To prove it by mathematical induction,

1. Show it is true for $n = k$

for $n = 1$,

$1^3 = \frac{1}{4} \times 1^2 \times 2^2$ is true

2. Assume it is true for $n = k$

$\therefore 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{1}{4} k^2 (k+1)^2$ is true (An assumption)

Now, prove it is true for "k+1"

$\therefore 1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \frac{1}{4} (k+1)^2 (k+2)^2$

We know that- $1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{1}{4} k^2 (k+1)^2$

(the assumption above)

So, we can do a replacement for all but the last term :-

$$\frac{1}{4}k^2(k+1)^2 + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$$

We know that

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{1}{4}k^2(k+1)^2$$

So, we can get by multiplying both sides by 4 :-

$$k^2(k+1)^2 + 4(k+1)^3 = (k+1)^2(k+2)^2$$

Let us take common $(k+1)^2$ from all factors we get :-

$$k^2 + 4(k+1) = (k+2)^2$$

And simplifying, we get :-

$$k^2 + 4k + 4 = k^2 + 4k + 4$$

They are the same ! So, it is true.

So, $1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$

is True. Proved.