BCA - Ist Sem.

Mathematical Induction :-

Mathematical induction is a concept in mathematics that is used to prove various mathematical statements and theorems. The principle of mathematical induction is sometimes referred to as PMI.

It is a technique used to prove basic theorems in mathematics that involve solving upto n finite natural terms.

For example, it can be used to prove sum of first n natural numbers is given by the formula $n(n+1)/2$.

Suppose $P(n)$ is a statement for n natural numbers, then it can be proved using the principle of Mathematical induction as :-

Formula is :- $\boxed{1+2+3+\cdots+n = \frac{n(n+1)}{2}}$

Step-by-Step Induction Proof :-

1. Base Case when $n=1$

$LHS = 1$

$RHS = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

Since $LHS = RHS$, the formula is true for $n=1$

2. Inductive Hypothesis :- Let us assume for $n=k$ where $k > 0$ (integer which is positive) then,

$1+2+3+\cdots+k = \frac{k(k+1)}{2}$ (This is our assumption)

3. Inductive Step :- Prove for $n = k+1$

we need to show that

$1+2+3+\cdots+k+(k+1) = \frac{(k+1)((k+1)+1)}{2}$

Start with the sum upto $k+1$:-

$(1+2+3+\cdots+k)+(k+1)$

Substitute the assumption $1+2+3+\cdots+k = \frac{k(k+1)}{2}$:-

$$\frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

After simplifying the expression :-

$$(k+1)\left(\frac{k+2}{2}\right) = \frac{(k+1)(k+2)}{2}$$

Hence we conclude :-

Since the formula holds for $n=1$, and assuming it holds for $n=k$ proves it holds for $n=k+1$, by the principle of mathematical induction,

the formula

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$ is true for

all natural numbers n.

Example :- Prove that $3^n - 1$ is multiple of 2.

<u>Proof</u> — Let us put $n=1$, then

$$3^n - 1 = 3^1 - 1 = 3 - 1 = 2$$

$\therefore$ 2 is a multiple of 2. Hence $3^n - 1$ is true for $n=1$

Now assume that it is true for $n=k$

$\therefore$ $3^k - 1$ is true.

Now, prove that $3^{k+1} - 1$ is a multiple of 2

3^{k+1} is also 3×3^k

And then split $3\times$ into $2\times$ and $1\times$

$$3^{k+1} - 1 = 3 \times 3^k - 1$$

$$= (2+1) \times 3^k - 1$$

$$= 2 \times 3^k + 3^k - 1$$

and it is multiple of 2 because 2×3^k is a multiple of 2.

$3^k - 1$ is true (as above assumption)

So, $3^{k+1} - 1$ is true proved.
for multiple of 2.

The principle of Mathematical Induction –

A set of positive integers that contains the integer 1 and the integer $n+1$ whenever it contains n must be the set of all positive integers.

Proof –

Let S be a set of positive integers containing the integer 1 and the integer $n+1$ whenever it contains n.

Assume that S is not the set of all positive integers.

Therefore, there are some +ve integers not contained in S.

By the well-ordering property, since the set of +ve integers not contained in S is non-empty, there is a least positive integer n which is not in S. Note that $n \neq 1$, since 1 is in S. Now since $n > 1$, the integer $n-1$ is a +ve integer smaller than n, and hence must be in S. But since S contains $n-1$, it must also contain $(n-1)+1 = n$, which is a contradiction, since n is supposedly the smallest +ve integer not in S. This shows that S must be the set of all +ve integers.

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