

(iii) [Equations Solvable for  $x$ .]

- If  $x$  can be expressed in terms of  $y$  and  $p$ , then the equation  $x = f(y, p)$  is said to be solvable for  $x$ . . . . . (1)

Differentiating w.r.t  $y$ , we get

$$\frac{dx}{dy} = \frac{1}{p} = F(y, p, \frac{dp}{dy}) . . . . . (2)$$

which is an equation of the first order and the first degree. Let the solution of (2) be

$$\phi(y, p, c) = 0 . . . . . (3)$$

Then eliminating  $p$  from (1) and (3), we get the primitive of the given equation.

- If elimination is not possible then values of  $x$  and  $y$  expressed in terms of parameter  $p$  together constitute the solution of the equation
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Q. 2. Solve:  $y = 2px + p^2y$

Solution:— The given equation can be put for  $x$  in the following way

$$2px = y - p^2y \Rightarrow x = \frac{y}{2p} - \frac{p^2y}{2}$$

$$\Rightarrow 2x = -py + \frac{y}{p}$$

Differentiating w.r.t  $y$ , we get

$$2 \frac{dx}{dy} = -p - y \frac{dp}{dy} + \frac{1}{p} - \frac{1}{p^2} \cdot \frac{dp}{dy}$$

$$\Rightarrow \frac{2}{p} - \frac{1}{p} + p = -y \left(1 + \frac{1}{p^2}\right) \frac{dp}{dy}$$

$$\Rightarrow \frac{1}{p} + p = -y \left(1 + \frac{1}{p^2}\right) \frac{dp}{dy}$$

$$\Rightarrow \left( \frac{1+p^2}{p} \right) + y \left( \frac{1+p^2}{p^2} \right) \frac{dp}{dy} = 0$$

$$\Rightarrow y \cdot \frac{dp}{dy} + \frac{1+p^2}{p} \times \frac{p^2}{1+p^2} = 0$$

$$\Rightarrow y \frac{dp}{dy} + p = 0 \Rightarrow y \frac{dp}{dy} = -p$$

$$\Rightarrow y dp = -p dy \Rightarrow \frac{dp}{p} + \frac{dy}{y} = 0$$

Integrating,  $\log p + \log y = \log C$

i.e.  $p y = C \quad \therefore \boxed{p = C/y}$

Putting this

Value of  $p$  in the given equation, we get

$$y = 2x\left(\frac{C}{y}\right) + \frac{C^2}{y^2} \cdot y = \frac{2xyC}{y} + \frac{C^2}{y}$$

i.e.  $\boxed{y^2 = 2xy + C^2}$  — Answer

Q2 Solve:  $y = 2px + y^2 p^3$

Solution:— Solving for  $x$ , we get

$$2px = -y^2 p^3 + y$$

$$\Rightarrow 2x = -y^2 p^2 + \frac{y}{p}$$

Differentiating w.r.t  $y$ , we get

$$2 \cdot \frac{dx}{dy} = -(2y \cdot p^2 + 2p y^2 \cdot \frac{dp}{dy}) + \left( \frac{1}{p} - \frac{1}{p^2} \frac{dp}{dy} \right)$$

$$\Rightarrow \boxed{\frac{2}{p} - \frac{1}{p^2} = -2py(p + y \frac{dp}{dy}) - \frac{1}{p^2} \frac{dp}{dy}}$$

$$\Rightarrow \frac{1}{p} + 2y p^2 + \frac{y dp}{p dy} \left( \frac{1}{p} + 2p^2 y \right) = 0$$

$$\Rightarrow \left( \frac{1}{p} + 2y p^2 \right) \left( 1 + \frac{y dp}{p dy} \right) = 0$$

Neglecting the first factor, we get

$$1 + \frac{dy}{p} \frac{dp}{dy} = 0 \quad \text{i.e. } \frac{dp}{p} + \frac{dy}{y} = 0$$

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Integrating,  $\log p + \log y = \log c$

$$\text{i.e. } py = c \Rightarrow p = \frac{c}{y}$$

Putting this value of  $p$  in the given equation,  
we get

$$y = 2x \frac{c}{y} + y^2 \cdot \frac{c^3}{y^3} = \frac{2xc}{y} + \frac{c^3}{y}$$

$$\text{i.e. } \boxed{y^2 = 2xc + c^3} \rightarrow \text{Answer.}$$

Q(3) Solve  $x = y + p^2$

Solution: — Given equation be  $x = y + p^2$

Solving for  $x$ .

Differentiating the given equation w.r.t.  $y$ , we get

$$\frac{1}{p} = 1 + 2p \frac{dp}{dy} \Rightarrow \frac{dp}{dy} = \left(\frac{1}{p} - 1\right)/2p = \frac{1-p}{2p^2}$$

$$\Rightarrow \left(\frac{2p^2}{1-p}\right) dp = dy \Rightarrow -2\left(\frac{p^2-1+1}{p-1}\right) dp = dy$$

$$\Rightarrow dy = -2\left[p+1 + \frac{1}{p-1}\right] dp$$

Integrating, we get

$$y = -2\left[\frac{p^2}{2} + p + \log(p-1)\right] + C$$

$$\text{i.e. } y = \left[-p^2 - 2p - 2\log(p-1)\right] + C$$

$$\text{or, } y = C - [p^2 + 2p + 2\log(p-1)] \quad (1)$$

Putting the value of  $y$  in the given equation,  
we get

$$x = C - [2p + 2\log(p-1)] \quad (2)$$

Eqs. (1) and (2) constitute  
the required solution.

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