

(ii) [Equations Solvable for  $y$ .]

If equation is solvable for  $y$ , we can express  $y$  in terms of  $x$  and  $p$ . i.e.

$$y = f(x, p) \quad \dots \dots \dots \quad (1)$$

Differentiating w.r.t  $x$ , we get

$$\frac{dy}{dx} = p = F(x, p, \frac{dp}{dx}) \quad \dots \dots \dots \quad (2)$$

which is an equation of the first order and the first degree. Let the solution of (2) is

$$\phi(x, p, c) = 0 \quad \dots \dots \dots \quad (3)$$

Then eliminating  $p$  from (1) and (3), we get the required solution.

- If  $p$  can't be easily eliminated then express values of  $x$  and  $y$  in terms of parameter  $p$  in the following form

$$x = \phi_1(p, c), \quad y = \phi_2(p, c)$$

These two relations together give the complete solution of the given equation.

Q. (i) Solve:  $y = 2px - p^2$

Solution:- Here, we see that the given eqn. be solvable for  $y$ , since the R.H.S. of the given eqn. is a function of  $x$  and  $p$ .

Now, differentiating the given equation w.r.t  $x$ , we get

$$p = 2p + 2x \frac{dp}{dx} - 2p \frac{dp}{dx}$$

$$\Rightarrow -p = 2(x-p) \frac{dp}{dx}$$

$$\Rightarrow p \frac{dx}{dp} + 2x - 2p = 0$$

$$\Rightarrow \frac{dx}{dp} + \left(\frac{2}{p}\right)x = 2 \quad \dots \dots \dots (1)$$

which is linear equation in  $x$ .

I.F. =  $e^{\int \frac{2}{p} dp} = e^{2 \int \frac{1}{p} dp} = e^{2 \log p} = e^{\log p^2} = p^2$

$\therefore \boxed{I.F. = p^2}$

$\therefore$  solution of (1) be

$$x \cdot p^2 = \int [2p^2] dp + C = C + \frac{2}{3} p^3$$

$$\therefore x = C p^{-2} + \frac{2}{3} p \quad \dots \dots \dots (2)$$

Also putting this value of  $x$  in the given eqn.

$$y = 2p \left( C p^{-2} + \frac{2}{3} p \right) - p^2$$

$$\Rightarrow y = 2C p^{-1} + \frac{4}{3} p^2 - p^2 \quad \dots \dots \dots (3)$$

$$\therefore y = 2C p^{-1} + \frac{1}{3} p^2$$

Eqs. (2) and (3) together constitute the general solution of given equation.

Q. (2) Solve  $y = 2px + p^2$

Solution:- Proceed same process as Q. (1).

Q. (3) Solve:  $p^2 - px + x = 0$

Solution:- The given equation can be put as

$$px = x + p^2 \Rightarrow y = x \cdot \frac{1}{p} + p$$

Differentiating w.r.t.  $x$ , we get

$$p = \frac{1}{p} + x \left(-\frac{1}{p^2}\right) \frac{dp}{dx} + \frac{dp}{dx}$$

$$\Rightarrow \left(p - \frac{1}{p}\right) = \left(1 - \frac{x}{p^2}\right) \frac{dp}{dx}$$

$$\Rightarrow \left(p - \frac{1}{p}\right) \frac{dx}{dp} + \left(\frac{1}{p^2}\right)x = 1$$

$$\Rightarrow \frac{dx}{dp} + \frac{1}{p(p^2-1)} \cdot x = \frac{p}{p^2-1} \quad \dots \dots \dots (1)$$

which is linear eqn. in  $x$  and  $p$ .

$$\therefore \int \frac{dp}{p(p^2-1)} = - \int \frac{dp}{p(1-p)(1+p)} = - \int \left\{ \frac{1}{p} + \frac{1}{2(1-p)} - \frac{1}{2(1+p)} \right\} dp$$

$$= - \left[ \log p - \frac{1}{2} \log(1-p) - \frac{1}{2} \log(1+p) \right]$$

$$= - \log \frac{p}{\sqrt{1-p^2}}$$

$$\therefore I.F. = e^{- \int \frac{dp}{p(p^2-1)}} = e^{- \log \frac{p}{\sqrt{1-p^2}}} = \frac{\sqrt{1-p^2}}{p}$$

Hence solution of (1) be

$$x \cdot \frac{\sqrt{1-p^2}}{p} = \int \left[ \frac{p}{p^2-1} \times \frac{\sqrt{1-p^2}}{p} \right] dp + C$$

$$= C - \int \frac{1}{\sqrt{1-p^2}} dp = C - \sin^{-1} p$$

$$\therefore x = \frac{p}{\sqrt{p^2-1}} (C - \sin^{-1} p)$$

Putting this value of  $x$  in the given equation, we find

$$y = p + \frac{p}{\sqrt{p^2-1}} (C - \sin^{-1} p) \cdot \frac{1}{p}$$

i.e. 
$$y = p + \frac{1}{\sqrt{p^2-1}} (C - \sin^{-1} p)$$

which is the required solution.

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